

# PhD Proposal: Lattice-based representation and reasoning for image understanding

Towards a new algebraic framework combining mathematical morphology, formal concept analysis, description logics, fuzzy sets and bipolarity

Labs :

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## Context

In this PhD project, we aim at addressing the challenging problem of high level scene understanding with a new methodological approach which integrates advantages of formal logics (description logics, formal concept analysis), and quantitative or semi-quantitative representations and reasoning in the image domain.

High level scene understanding is the task of inferring semantics from image contents in a form which is close to and suitable for application domain decision-making. This semantics cannot be considered as being included explicitly in the image itself but rather depends on prior knowledge on the domain and on the context of use of the image. Model-based image understanding is a subfield where semantics inference is guided by a model e.g. a graph or an ontology. In this context, modeling imperfections of knowledge and information is also a key aspect.

## Research program

In [2], we proposed to formulate model based scene understanding as an abductive reasoning process. A scene is viewed as an observation and the task of

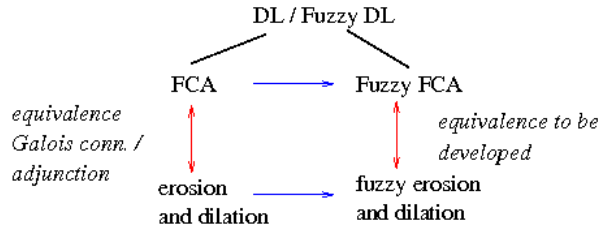


Fig. 1: Overview of the links between fuzzy set, mathematical morphology, formal concept analysis and description logics.

interpretation is considered as the “best” explanation considering the terminological knowledge part of a description logic about the scene context. From a theoretical standpoint, we provided some insights on the definition of abduction inference services in description logics based on mathematical morphology over concept lattices. The construction of these lattices is based on exploiting the advances of using formal concept analysis in description logics. Explanations are computed using morphological erosions on the underlying concept lattice. More recently [1], we continued our effort on unifying mathematical morphology and formal concept analysis. Beyond the classical relationship between the Galois connections defined by the derivation operators and the adjunctions underlying the algebraic mathematical morphology framework we defined mathematical morphology operators over concept lattices, based on distances, valuations, or neighborhood relations in concept lattices.

The PhD project builds upon this previous work and is aimed at developing a principled, formal, algebraic-based, framework unifying mathematical morphology, description logics, formal concept analysis, bipolarity and fuzzy sets. The overall objective is to derive in a natural way classical and non-classical reasoning tools while accounting for intrinsic properties of the spatial knowledge, including vagueness and bipolarity.

A first task will consist in extending our developments to the fuzzy case. Uncertainty in FCA has been addressed from fuzzy, possibilistic and rough sets perspectives [8, 3, 6]. Here we propose a morphological view, which encompasses all the others. Since a Galois connection is equivalent to erosion and dilation, fuzzy mathematical morphology leads directly to fuzzy FCA. Links with previous approaches will be explored, as well as the new properties inherited from the morphological framework. The diagram in Figure 1 summarizes the interactions and links to be developed in this PhD.

A second task concerns the integration of bipolarity in this framework. Bipolarity is an important feature of information processing, for differentiating between positive information (e.g. what is possible, observed, preferred) and negative information (e.g. constraints, what is impossible or forbidden). This feature is also present in spatial information processing. For instance positive position may refer to possible places for an object, while negative information may refer to forbidden places (e.g. because they are already occupied). However, bipolar-

ity has not been much considered in spatial reasoning so far. Although a few work uses interval-valued fuzzy sets in image processing, none exploits the true bipolarity nature of spatial information, where positive and negative parts may come for different sources, and cannot be simply represented as intervals, but have to be represented on two different and independent scales (up to a consistency constraint). Following the seminal work by D. Dubois and H. Prade [7] on this type of bipolarity and its representation in a possibilistic setting, we developed in [5, 4] a framework for dealing with bipolar fuzzy sets (hence accounting for both bipolarity and vagueness) in the spatial domain. In particular we have shown that once a complete lattice structure is defined, mathematical morphology operators on such mathematical objects can be developed, with good properties.

Although the formal algebraic framework is equivalent to other extensions of mathematical morphology, the nature of bipolar information has some specificities that deserve special attention. Thus several issues remain open, and some of them will be addressed in this PhD project. One important issue concerns the choice of a partial ordering on the bipolar fuzzy sets. Two extreme cases can be considered: Pareto ordering and lexicographic ordering, but also several other intermediate ones. An objective of this task is to derive meaningful orderings depending on the semantics of the positive and negative information. For instance Pareto ordering may be interesting if positive and negative information play symmetric ones, while lexicographic ordering strongly favors one of them. Another issue concerns the modeling of the bipolarity of spatial relations, which has not been addressed so far. For instance, some relations often go by pairs (e.g. left and right), while they are not the contrary of each other. Bipolar models can then be useful. Similarly bipolarity can be integrated in the FCA formalisms, and the mathematical morphology operators will apply directly thanks to the lattice structure.

## **Profile/skills**

We are looking for highly motivated students with an excellent degree (M.Sc., M.Eng. or equivalent) in Applied Mathematics or Computer Science with a demonstrable interest in universal algebra, logics and their applications in knowledge representation and reasoning. Interest in image understanding will be highly appreciated too.

## **Application instructions**

The application package must include a detailed CV, a motivation letter, a summary of the master thesis, two recommendations letters and a transcript of the grades. The application deadline is **May 15, 2014**.

## References

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- [3] R. Belohlávek and V. Vychodil. What is a fuzzy concept lattice. In *Proc. CLA*, volume 5, pages 34–45, 2005.
- [4] I. Bloch. Lattices of fuzzy sets and bipolar fuzzy sets, and mathematical morphology. *Information Sciences*, 181:2002–2015, 2011.
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- [8] Y. Lei and M. Luo. Rough concept lattices and domains. *Annals of Pure and Applied Logic*, 159(3):333–340, 2009.